Using Multivariate Dynamic Conditional Correlation GARCH model to analysis financial market data

Prepared

Fatma Y. Alshenawy
Applied statistics and insurance Department - Faculty of commerce
Mansoura university
felshinawy@gmail.com

Doaa A. Abdo
Applied statistics and insurance Department- Faculty of commerce
Mansoura university
doaaashour@mans.edu.eg

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Abstract:

In financial markets, understanding the dynamic relationships between assets is crucial for effective portfolio management. This study highlights the importance of using the DCC-GARCH (Dynamic Conditional Correlation - Generalized Autoregressive Conditional Heteroskedasticity) model as a powerful multivariate analysis tool to capture the dynamic correlations between the S&P 500, Crude Oil Price, Natural Gas Price, and Gold Price. The DCC-GARCH model provides a flexible framework for modeling time-varying correlations, allowing investors to account for the changing relationships between assets over time. The study estimates the correlations and forecasts their evolution over the next 365 days, providing valuable insights for portfolio optimization and risk management. The results demonstrate the potential diversification benefits offered by these assets and emphasize the need for adaptive portfolio management based on the dynamic correlations. By employing the DCC-GARCH model, investors can better understand the complex interactions between assets and make more informed decisions about asset allocation, ultimately leading to improved risk-adjusted returns. This study underscores the significance of incorporating advanced multivariate techniques, such as DCC-GARCH, in financial analysis and portfolio management.

Keywords: dynamic correlations, DCC-GARCH, multivariate analysis, time-varying correlations, portfolio management, financial markets, asset interactions, Quasi likelihood.

1-INTRODUCTION:

In recent years, financial markets have experienced significant fluctuations and increasing interdependence among asset classes. Understanding the dynamic relationships between various financial assets is crucial for investors and portfolio managers, as these relationships directly impact portfolio diversification, risk management, and asset allocation strategies. Traditional portfolio management approaches, based on constant correlations, may not adequately capture the complexities of contemporary financial markets. This study aims to explore the application of advanced techniques, such as the Dynamic Conditional Correlation - Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model, to better understand the evolving interactions between key financial assets and their implications for portfolio management.

The DCC-GARCH model, introduced by Engle (2002), has emerged as a popular method for modeling time-varying correlations between financial assets, providing a flexible framework for capturing the complex interactions between them. By accounting for dynamic correlations, investors can potentially adjust their asset allocations to optimize risk-adjusted returns and improve portfolio performance.

The main objective of this study is to investigate the dynamic correlations between the S&P 500, Crude Oil Price, Natural Gas Price, and Gold Price using the DCC-GARCH model. We seek to understand how these interdependencies evolve over time and their implications for portfolio diversification and risk management. Additionally, we aim to provide practical recommendations for investors and portfolio managers based on our findings.

This paper is structured as follows: Section 2 presents a literature review of relevant studies on dynamic correlations, DCC-GARCH, and portfolio management. Section 3 describes the model specification. Section4 describes the data sources and variables. Section 5 presents the results and discusses the practical implications of our results for portfolio management as well as the limitations of our study and potential areas for future research. Finally, Section 6 concludes the paper by summarizing the main findings and their significance for investors and portfolio managers.

2-Literature Review:

Understanding the dynamic relationships between various financial assets is crucial for effective portfolio management. Studies have shown that asset correlations can change over time and are often influenced by macroeconomic factors, market conditions, and investor sentiment (Ang & Bekaert, 2002; Longin & Solnik, 2001). Dynamic correlations can have significant implications for portfolio diversification, risk management, and asset allocation strategies (Bekaert, Hodrick, & Zhang, 2009).

Before diving into the DCC-GARCH model, it is important to briefly mention the univariate GARCH model introduced by Bollerslev (1986). The GARCH model extends the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982) and allows for the modeling of the volatility clustering observed in financial time series. The GARCH model has been extended to a multivariate setting, known as the Multivariate GARCH

(MGARCH) model (Bauwens et al., 2006). A popular MGARCH model is the Constant Conditional Correlation (CCC) model introduced by Bollerslev (1990). In this model, the conditional correlation between any two variables is assumed to be constant over time. However, this assumption is often too restrictive for financial time series, as conditional correlations tend to change over time.

To address the limitations of the CCC model, Engle (2002) proposed the DCC-GARCH model. The DCC-GARCH model allows for time-varying conditional correlations while maintaining the simplicity and tractability of the CCC model. The DCC-GARCH model decomposes the conditional covariance matrix into conditional standard deviations and conditional correlations. This decomposition allows for the separate modeling of volatilities and correlations, enabling more flexibility and ease of interpretation.

The Dynamic Conditional Correlation - Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model, introduced by Engle (2002), has emerged as a popular method for modeling time-varying correlations between financial assets. The model allows for the estimation of conditional correlations that evolve over time, providing a flexible framework for capturing the complex interactions between assets (Engle, 2002; Silvennoinen & Teräsvirta, 2009).

The incorporation of dynamic correlations in portfolio management has been shown to improve portfolio performance and risk management (Alexander, 2008; Tse& Tsui, 2002). By accounting for time-varying correlations between assets, investors can better understand the potential diversification benefits and adjust their asset allocations accordingly (Kroner & Ng, 1998; Markowitz, 1952). Several studies have applied the DCC-GARCH model to portfolio optimization and found improved risk-adjusted returns compared to traditional methods (Bollerslev, 1990; Chua, Suardi, &Tsiaplias, 2017).

This literature review highlights the importance of understanding dynamic correlations in financial markets, the advantages of the DCC-GARCH model for modeling time-varying correlations, and the potential benefits of incorporating dynamic correlations in portfolio management. Future research could explore the application of the DCC-GARCH model to other asset classes, the impact of macroeconomic factors on dynamic correlations, and the development of new methods for portfolio optimization based on time-varying correlations.

3- Model Specification:

Many studies of financial econometrics are concerned with analyzing fluctuations that change with time. Engle (1982) presented the autoregressive conditional heteroscedasticity (ARCH) model, which was generalized by Bollerslev (1986) to include generalized autoregressive conditional heteroscedasticity models. (GARCH), which allows conditional variances to vary with time through a function of prior variances.

The univariate GARCH(p,q) model takes the following form:

$$\boldsymbol{r_t = \mu_t + \varepsilon_t,} \qquad \textbf{(1)}$$

$$\boldsymbol{\varepsilon_t = h_t^{1/2} z_t,} \tag{2}$$

$$\boldsymbol{h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}\ .} \tag{3}$$

Where:

$\boldsymbol{r_t}$: logarithm of returns over timet

$\boldsymbol{\varepsilon_t}$ **:**The residuals are assumed to be an unconnected series, meaning that when exposed to large fluctuations, this does not necessarily mean that the value of ε_tt is large, but only means that the probability of obtaining a greater value of ε_t is increased.

$\boldsymbol{\mu_t}$: The expected value of conditional return $\mathbf{r_t}$, which can be put in the form of a time series ARMA or as a constant value.

$\boldsymbol{h_t}$: The square of the fluctuations or, in other words, the conditional variations at time t. It can be rephrased in the form:

$$\boldsymbol{h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}.} \tag{4}$$

$\mathbf{z_t}$: A series of independent random variables which follows a normal distribution N (0,1)

With the wide application of univariate GARCH models, there has been a need for development in its application to include multivariate M-GARCH models (Elisabth, (2009)), as they represent more reliable models when predicting the movements of the returns of financial assets, which is important when pricing financial assets in a portfolio, since financial fluctuations move together over time across financial markets.

MGARCH models explain how covariances move through time by constructing a covariance matrix. Accordingly, the multivariate M-GARCH models help to making better decisions in different areas of financial market applications.

Bollerslev (1986) presented the first M-GARCH model which takes the following structure:

$$\boldsymbol{r_t = \mu_t + \varepsilon_t} \quad \textbf{(5)}$$

$$\boldsymbol{\varepsilon_t = H_t^{1/2} z_t} \quad \textbf{(6)}$$

where:

$\boldsymbol{r_t}$**:** is the n×1 vector of the logarithms of n variables at time t

$\boldsymbol{\varepsilon_t}$: Residue vector n×1 for n variables at time t, with mean$\mathrm{E}(\varepsilon_t) = 0$and covariance matrix$\mathrm{cov}(\varepsilon_t) = \mathrm{H}_t$

$\boldsymbol{\mu_t}$: An n×1 vector of the expected values of the conditional returns of n variables at time t.

$\boldsymbol{H_t}$:n×n matrix of conditional variances of $\varepsilon_t$at time t.

$\boldsymbol{z_t}$:An n×1 vector of random errors that follows a normal distribution with a mean$\mathrm{E}(z_t) = 0$ ، $\mathrm{E}(z_t z_t^T) = 1$

This study is concerned with the use of models of conditional variances and correlations. The main idea of this class of models is to divide the matrix of conditional variances into two parts: the conditional standard deviations and the conditional correlations as follows:

$$H_t = D_t R_t D_t \tag{7}$$

where D_t represents the diagonal conditional standard deviation matrix on the structure:

$$D_t = diag(h_{1t}^{1/2}, \dots, h_{nt}^{1/2}), \tag{8}$$

Whereas R_t represents the matrix of conditional correlations. The models of this category can be classified into two groups: the first with a fixed correlation matrix, and the second when the correlation matrix is variable with time, which is our concern in this paper.

(3-1) Time-varying conditional correlation model:

It is also called the Dynamic Conditional Correlation (DCC-GARCH) model, which was proposed by Engle (2002). It is a non-linear combination of univariate GARCH models, and it is also a general picture of CCC-GARCH models and can be formulated as follows:

$$H_t = D_t R_t D_t, \tag{9}$$

Where the conditional standard deviation matrix can be expressed as:

$$D_t = diag(h_{11t}^{1/2}, \dots, h_{NNt}^{1/2}) \tag{10}$$

Or in matrix form:

$$D_t = \begin{bmatrix} \sqrt{h_{11t}} & 0 \; \dots & & 0 \\ 0 & \sqrt{h_{22t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sqrt{h_{NNt}} \end{bmatrix} \tag{11}$$

Each element h_{iit} can be expressed in a univariate GARCH form as follows:

$$\boldsymbol{h_{it} = \alpha_{i0} + \sum_{q=1}^{Q} \alpha_{iq} \varepsilon_{i,t-q}^{(2)} + \sum_{p=1}^{P} B_{ip} h_{i,t-p}} \quad \textbf{(12)}$$

The univariate GARCH model can take different orders, and the simple first-order GARCH (1,1) model is usually used

The matrix $\mathbf{R_t}$ represents the correlation matrix of standard errors $\mathbf{u_t}$, and the conditional correlation matrix R_t takes the form of a symmetric matrix as follows:

$$\boldsymbol{R_t} = \begin{bmatrix} \mathbf{1} & \boldsymbol{\rho}_{\mathbf{12,t}} \boldsymbol{\rho}_{\mathbf{13,t}} & \cdots & \boldsymbol{\rho}_{\mathbf{1n,t}} \\ \boldsymbol{\rho}_{\mathbf{12,t}} & \mathbf{1} \quad \boldsymbol{\rho}_{\mathbf{23,t}} & \cdots & \boldsymbol{\rho}_{\mathbf{2n,t}} \\ \boldsymbol{\rho}_{\mathbf{13,t}} \boldsymbol{\rho}_{\mathbf{13,t}} & \mathbf{1} & \ddots & \vdots \\ \vdots\vdots \quad \ddots & & \ddots & \boldsymbol{\rho}_{\mathbf{n-1,n,t}} \\ \boldsymbol{\rho}_{\mathbf{1n,t}} \boldsymbol{\rho}_{\mathbf{2n,t}} & \cdots \; \boldsymbol{\rho}_{\mathbf{n-1,n,t}} & & \mathbf{1} \end{bmatrix} \quad \textbf{(13)}$$

When formulating the conditional correlation matrix R_t, two basic conditions must be ensured:

1- The covariance matrix H_t is a positive definite matrix, and then $R_t$must be a positive definite matrix.

2- All elements of the conditional correlation matrix $R_t$must be less than or equal to one.

To make sure that the two previous conditions are met in the conditional correlation matrix, it can be divided as follows:

$$\boldsymbol{R_t = Q_t^{*-1} Q_t Q_t^{*-1}}, \quad \textbf{(14)}$$

where $Q_t = (q_{itj})$ represents an N×N symmetric positive definite matrix and takes the following form:

$$\boldsymbol{Q_t = (1-\alpha-\beta)\bar{Q} + \alpha u_{t-1}\acute{u}_{t-1} + \beta Q_{t-1}},\qquad \textbf{(15)}$$

where $\boldsymbol{\bar{Q} = cov(u_t\acute{u}_t) = E(u_t\acute{u}_t)}$ is an N×N unconditional covariance matrix ($u_t$unconditional covariance matrix)

Where $\boldsymbol{u_t = \varepsilon_{it}/\sqrt{h_{iit}}}$ represents the standard error, while $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ represent non-negative parameter scalars, so that $\alpha + \beta < 1$ to ensure that the covariance matrix is positive.

$Q_t^{*-1} = diag(q_{11t}^{\frac{1}{2}}, \dots, q_{NNt}^{\frac{1}{2}})$ also represents a diagonal matrix with the square root elements of the main diagonal of the matrix Q_{t}.

(3-2) Model estimation using Quasi-Maximum Likelihood(QML)

The method of Quasi-Maximum Likelihood is one of the most common methods in estimating the matrix of conditional covariances of MGARCH general autoregressive models.

Assuming that:

$H_{\mathrm{t}}(\theta)$: represents N×N matrix which is the positive definite matrix of the conditional covariances of the remainder vector ε_{t}.

F_{t} : represents the information available at time t,

Therefore

$$\boldsymbol{E_{t-1}[\varepsilon_t | F_{t-1}] = 0}\ ;\qquad \textbf{(16)}$$

$$\boldsymbol{E_{t-1}[\varepsilon_t \grave{\varepsilon}_t | F_{t-1}] = H_t(\theta)}.\qquad \textbf{(17)}$$

Assuming θ_0 represents the feature vector to be estimated given T from the sample observations, QML estimates for the feature vector $\theta_0$can be obtained by maximizing the function of the Gaussian log likelihood function, defined as follows:

$$logL_T(\theta) = -\frac{N.T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} log|H_t| - \frac{1}{2}\sum_{t=1}^{T} \acute{\varepsilon}_t H_t^{-1}\varepsilon_t, \qquad \textbf{(18)}$$

Assuming that the time series under study is a static series (Wenjing and Yiyu, 2010), and that the residuals follow the Conditional Gaussian distribution

4-Data Description:

The data used in this paper consists of four key financial assets: the S&P 500 Index, Crude Oil Price, Natural Gas Price, and Gold Price. To conduct our analysis, we utilize daily data from May15, 2010, to May15, 2023 (3275 observations). The data were obtained from macrotrends website (https://www.macrotrends.net/) and included four major econometric variable as:

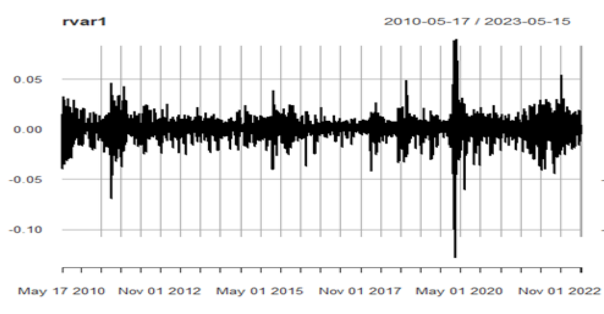
1. S&P 500 Index: The S&P 500 Index is a widely recognized benchmark for the U.S. stock market, comprising 500 leading companies listed on U.S. stock exchanges. The index is market-capitalization-weighted, reflecting the performance of large-cap U.S. equities. We obtain daily closing price data for the S&P 500 Index from Yahoo Finance.

2. Crude Oil Price: Crude oil is a critical global commodity, and its price movements can have significant implications for financial markets and the global economy. We use the West Texas Intermediate (WTI) Crude Oil Price as a proxy for the global crude oil market. We obtain daily closing price data for WTI Crude Oil from the U.S. Energy Information Administration (EIA).

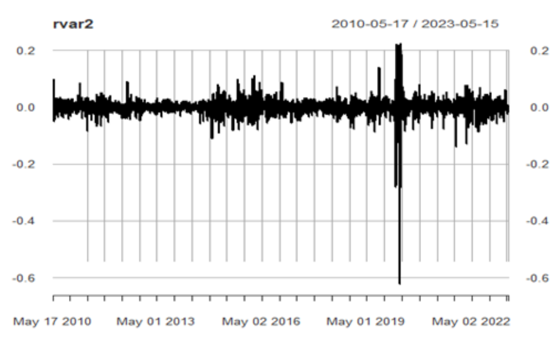
3. Natural Gas Price: Natural gas is another important energy commodity, and its price dynamics can influence various sectors of the economy. We use the Henry Hub Natural Gas Spot Price as a representative for the natural gas market. We obtain daily closing price data for the Henry Hub Natural Gas from the U.S. Energy Information Administration (EIA).

4. Gold Price: Gold is often considered a safe-haven asset and has historically played a significant role in global financial markets. We use the London Bullion Market Association (LBMA) Gold Price, which is widely regarded as the global benchmark for gold prices. We obtain daily closing price data for gold in USD per troy ounce from the LBMA website.

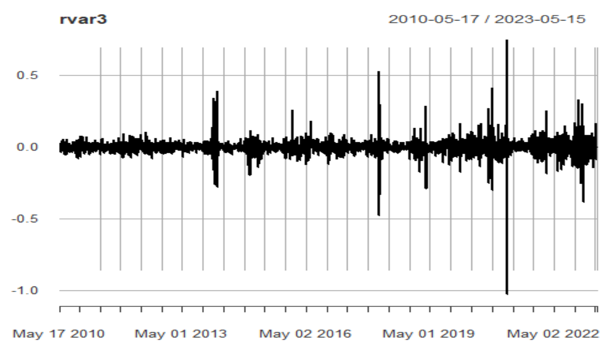
To ensure the reliability and consistency of our analysis, we preprocess the data by adjusting for potential outliers, missing values, and non-stationarity. We apply natural logarithms to the price series and calculate daily log returns to stabilize the variance and achieve stationarity. This transformation allows us to focus on the changes in asset prices rather than their absolute levels.



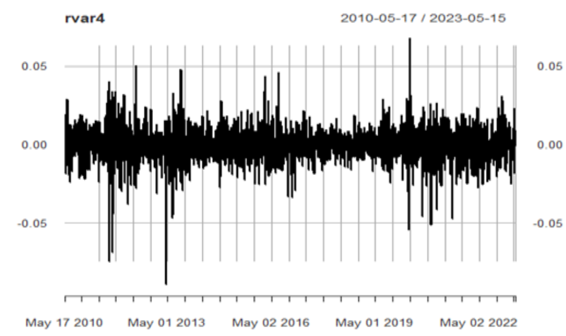
S & P 500 index



Cured oil prices



Natural gas prices



Gold prices

Figure[1] Plot of returns series of variables

The time series plots shown in figures [1] for the S & P 500 Index, Crude Oil Price, Natural Gas Price, and Gold Price exhibit varying degrees of volatility and distinct periods of significant price movements. These patterns suggest that the dynamics of these assets are influenced by various factors, such as macroeconomic conditions, market events, and investor sentiment.

In our investigation, we analyze the log returns of these four financial assets using the DCC-GARCH model to estimate their time-varying correlations. This approach enables us to better understand the evolving interdependencies between these assets and their implications for portfolio diversification and risk management.

5- Empirical Results:

(5-1) Descriptive summary

Table (1) presents summary of descriptive statistics analysis of the S&P 500 Index, Crude Oil Price, Natural Gas Price, and Gold Price provides valuable insights into the characteristics of these financial assets over the study period. It shows that the average value of the S&P 500 Index is 2,482.68, with a standard deviation of 1,001.47, indicating substantial variability in the index over time. The index ranges from a minimum of 1,022.58 to a maximum of 4,796.56, reflecting the significant market fluctuations experienced during the study period, while the mean Crude Oil Price is 71.31 USD per barrel, with a standard deviation of 22.71, suggesting considerable volatility in oil prices. The minimum and maximum prices are 11.26 and 123.70 USD, respectively, highlighting the wide range of price movements in the crude oil market.

Also, the average Natural Gas Price is 3.43 USD per million British thermal units (MMBtu), with a standard deviation of 1.37, indicating a moderate degree of price volatility. The price ranges from a minimum of 1.33 to a maximum of 23.86 USD, illustrating the significant price fluctuations in the natural gas market. and finally, the mean Gold Price is 1,466.69 USD per troy ounce, with a standard deviation of 250.87, reflecting a relatively lower level of volatility compared to the other assets in our analysis. The minimum and maximum gold prices are 1,049.60 and 2,058.40 USD, respectively, demonstrating the range of gold price movements over the study period.

Table (1): Descriptive statistics

	Mean	**Standard Deviation**	**minimum**	**First Quartile**	**median**	**maximum**	**ADF**	**p-value**
S & p500	2482.67	1001.46	1022.580	1680.550	2185.79	4796.56	-2.923	0.187*
cued oil price	71.3087	22.7061	11.258	51.600	70.20	123.70	-1.9068	0.617*
natural gas price	3.43051	1.36872	1.330	2.630	3.06	23.86	-3.5537	0.0037
gold price	1466.68	250.874	1049.600	1255.375	1358.70	2058.40	-1.6519	0.725*

Note:* denotes statistical significance at the 1% level

Based on the Augment Dickey-Fuller (ADF) test results, we conclude that the Natural Gas Price series is stationary, while the S&P 500 Index, Crude Oil Price, and Gold Price series are non-stationary. To address the non-stationary nature of these series, appropriate transformations, such as differencing or computing log returns, should be applied before conducting further analyses. This step is crucial to ensure the validity of the results and avoid spurious relationships in subsequent econometric modeling.

(5-2) Estimation model

We estimated a Dynamic Conditional Correlation (DCC) GARCH model to examine the time-varying correlations between the returns of four assets in our dataset: S&P 500, crude oil price, natural gas price, and gold price. The multivariate time series consists of 3,275 observations for each asset. Each asset is modeled using a univariate GARCH(1,1) process with an ARMA(1,1) mean equation and multivariate normal errors. The DCC-GARCH model captures the dynamic correlations between the assets by applying a DCC(1,1) structure to the conditional correlations.

For each asset \`i\`, the conditional mean equation is given by:

$$\boldsymbol{r_{\{i,t\}} = \mu_i + \varphi_i * \left(r_{\{i,t-1\}} - \mu_i\right) + \theta_i * \varepsilon_{\{i,t-1\}} + \varepsilon_{\{i,t\}}} \tag{19}$$

where $r_{\{i,t\}}$ represents the return of asset \`i\` at time \`t\`, μ_i is the constant mean, φ_i and \`$\theta_i$ are the AR and MA coefficients, respectively, and \`ε_{i,t}\`$\varepsilon_{\{i,t\}}$ is the error term.

The univariate GARCH(1,1) model for each asset \`i\` is specified as:

$$\boldsymbol{h_{\{i,t\}} = \omega_i + \alpha_i * \varepsilon_{\{i,t-1\}}{}^2 + \beta_i * h_{\{i,t-1\}}} \tag{20}$$

where $h_{\{i,t\}}$ denotes the conditional variance of asset `i` at time `t`, ω_i is the intercept, α_i is the ARCH coefficient, and β_i is the GARCH coefficient.

The DCC structure is given by:

$$\boldsymbol{q_t = (1 - a - b) * Q + a * (\varepsilon_{\{t-1\}} * (\grave{\varepsilon}_{\{t-1\}})) + b * q_{\{t-1\}}} \qquad \textbf{(21)}$$

where q_t is the matrix of dynamic conditional correlations at time `t`, `Q` is the unconditional correlation matrix, `a` and `b` are the DCC coefficients, and ε_t is the vector of standardized residuals.

Table (2): The estimated DCC-GARCH model

var	parameters	estimate	Std. Error	t value	Pr(>|t|)
S & P 500	μ_1	2.0875e+03	4.558665	457.923930	0.000000
	ω_1	2.0941e+02	44.166874	4.741229	0.000002
	α_1	9.9863e-01	0.063367	15.759556	0.000000
	β_1	0.0000e+00	0.063103	0.000000	1.000000
Cured Oil price	μ_2	5.6391e+01	0.282345	199.723818	0.000000
	ω_2	8.7513e-01	0.217479	4.023986	0.000057
	α_2	9.1681e-01	0.050904	18.010538	0.000000
	β_2	8.2187e-02	0.050054	1.641964	0.100598
Natural Gas price	μ_3	2.8188e+00	0.014233	198.049832	0.000000
	ω_3	6.1400e-03	0.000901	6.816741	0.000000
	α_3	9.9900e-01	0.019670	50.787404	0.000000
	β_3	0.0000e+00	0.000976	0.000242	0.999807
Gold price	μ_4	1.2880e+03	2.815181	457.524694	0.000000
	ω_4	7.8636e+01	20.705568	3.797803	0.000146
	α_4	9.3290e-01	0.043677	21.358797	0.000000
	β_4	6.6100e-02	0.047449	1.393093	0.163592
Joint dcca1	a	1.9417e-01	0.008920	21.768188	0.000000
Joint dccb1	b	8.0549e-01	0.008959	89.909159	0.000000

The table of optimal parameters provides the estimated values, standard errors, t-values, and p-values for each parameter in the DCC GARCH model for each of the four-time series (S&P 500, Crude Oil Price, Natural Gas Price, and Gold Price).

The mean μ (long-term average) of the time series. For example, the estimated mean for the S&P 500 Index is 2,087.5. The highly significant t-values and p-values close to 0 for all four series suggest that the mean estimates are statistically significant.

The constant term in the GARCH model (ω), representing the long-term average volatility. For example, the estimated omega for the S&P 500 Index is 209.41. The significant t-values and p-values close to 0 indicate that the omega estimates are statistically significant.

The GARCH term **(α_1),** representing the impact of past squared residuals (shocks) on current volatility. For example, the estimated alpha1 for the S&P 500 Index is 0.99863. The significant t-values and p-values close to 0 for most series suggest that past shocks have a significant impact on current volatility.

The ARCH term **(β_1),** representing the impact of past volatilities on current volatility. For example, the estimated beta1 for the S&P 500 Index is 0. In this case, beta1 is not statistically significant for most of the series, as reflected by the high p-values.

Finally, These are the DCC coefficients (Joint dcca1 and dccb1), representing the dynamics of the correlations among the four time series. The estimated dcca1 is 0.19417, and the estimated dccb1 is 0.80549. Both coefficients have highly significant t-values and p-values close to 0, suggesting that the dynamic correlations among the series are statistically significant.

(5-3) DCC Correlation structure:

The estimated dynamic conditional correlations at the last observation are shown in table (3)

Table(3): estimation of DC Correlation matrix

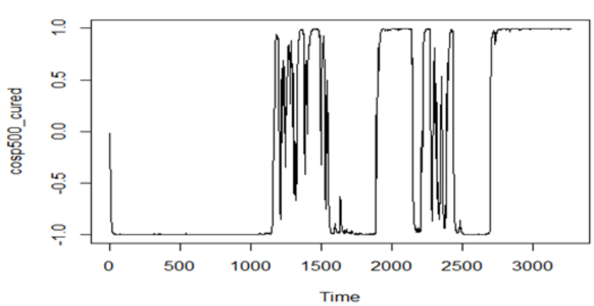
	S&P 500	Crude Oil Price	Natural Gas Price	Gold Price
S&P 500	**1.0000000**	**0.9958647**	**-0.9883149**	**0.9977875**
Crude Oil Price	**0.9958647**	**1.0000000**	**-0.9844202**	**0.9956341**
Natural Gas Price	**-0.9883149**	**-0.9844202**	**1.0000000**	**-0.9853399**
Gold Price	**0.9977875**	**0.9956341**	**-0.9853399**	**1.0000000**

The correlations reveal the relationships between the assets as a strong positive correlation between S&P 500 and Crude Oil Price of 0.9959 indicates that the returns of these two assets tend to move in the same direction. This suggests that investors may not achieve significant diversification benefits by investing in both assets simultaneously, while a strong negative correlation between S&P 500 and Natural Gas Price of -0.9883 implies that the returns of these two assets tend to move in opposite directions. This relationship may provide investors with an opportunity to diversify their portfolios and reduce overall risk. Also, a strong positive correlation between S&P 500 and Gold Price of 0.9978 indicates that the returns of these two assets tend to move in the same direction. Similar to the relationship between the S&P 500 and Crude

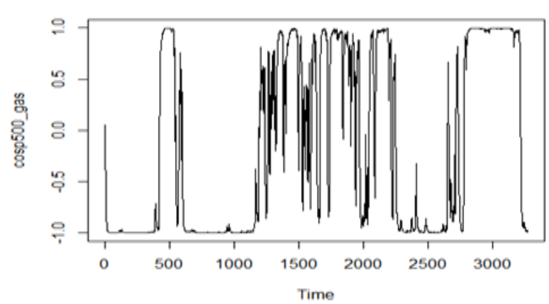
Oil Price, investing in both assets simultaneously may not provide significant diversification benefits.

Crude Oil Price and Natural Gas Price have a strong negative correlation of -0.9844 which suggests that the returns of these two assets tend to move in opposite directions. Investors may be able to achieve diversification benefits by investing in these assets simultaneously, while Crude Oil Price and Gold Price have a strong positive correlation of 0.9956 which indicates that the returns of these two assets tend to move in the same direction. This relationship may not offer significant diversification benefits for investors.

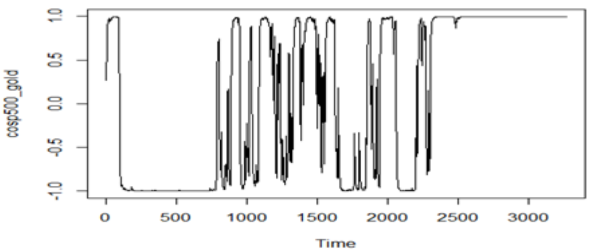
A strong negative correlation between Natural Gas Price and Gold Price of -0.9853 implies that the returns of these two assets tend to move in opposite directions. This relationship may provide investors with an opportunity to diversify their portfolios and reduce overall risk.



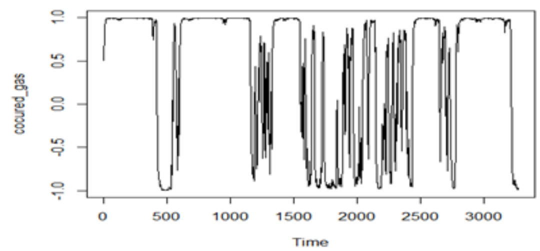
Dc correlation between s&p500 and cured oil price



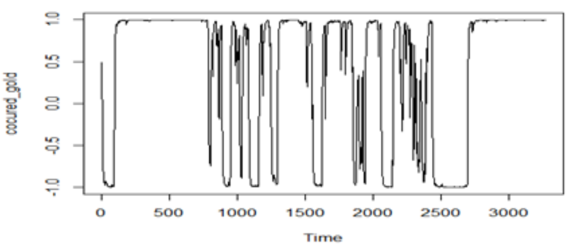
Dc correlation between s&p500 and natural gas price



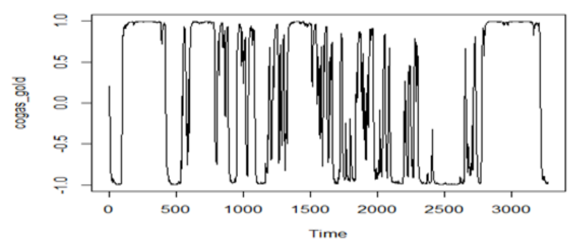
Dc correlation between s&p500 and gold price



Dc correlation between cured oil price and natural gas price



Dc correlation between cured oil price and gold price



Dc correlation between natural gas price and gold price

Figure[2] :DCC Correlation plot

It is important to note that the dynamic conditional correlations are time-varying, and the relationships between the assets can change over time. Investors should monitor these correlations and adjust their portfolios accordingly to maximize diversification and manage risk effectively. Also, we observe periods of high positive correlations, low or negative correlations, and varying degrees in between.

The estimated DCC coefficients are `a = 0.0047` and `b = 0.9939`. Both coefficients are statistically significant at the 1% level.

(5-4) DC Covariance structure:

The covariance matrix shows the degree to which the returns of the assets move together. The diagonal elements of the matrix represent the variance of each asset, while the off-diagonal elements represent the covariance between each pair of assets.

The estimated covariance matrix of the assets at the last observation is shown in:

Table (4) : Estimation of Dc covariance matrix

Variable	S&P 500	Crude Oil Price	Natural Gas Price	Gold Price
S&P 500	**4,142,109.623**	**44,888.35811**	**-1,253.94163**	**1,283,745.662**
Crude Oil Price	**44,888.358**	**490.50701**	**-13.59172**	**13,939.670**
Natural Gas Price	**-1,253.942**	**-13.59172**	**0.38864**	**-388.318**
Gold Price	**1,283,745.662**	**13,939.66991**	**-388.318**	**399,632.016**

The results in table (4) provide conditional covariance matrix from DCC-GARCH model. This matrix gives an idea of the most recent covariances between the four variables: S&P500, Cued. Oil. Price, Natural. Gas. price, and Gold. price. Covariance measures the joint variability of two variables, indicating how they move together.

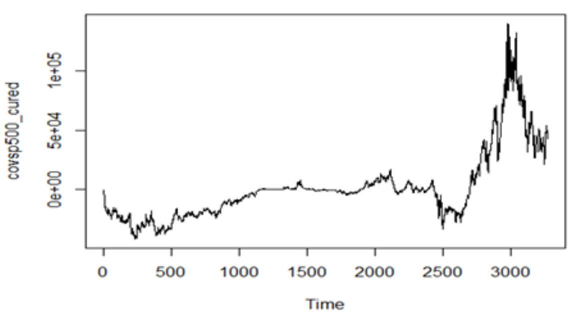
The covariance between both of s&p 500 and cued. oil. price is 44,888.35811. Also, covariance between s&p500 and gold. price is 1,283,745.6622, and the covariance between cued. oil. price and gold. price is 13,939.6699, indicating that both of these two variables tend to move together in the same direction, as the value is positive. This aligns with the strong positive correlation observed previously.

While, the covariance between both of: s&.p500 and natural. gas. price is -1,253.9416326,cued.oil.price and natural. gas. price is -13.5917165 and natural. gas. price and gold. price is -388.3178791, which shows that both of these two variables tend to move in opposite directions, as the value is negative. This is consistent with the negative correlation observed earlier.

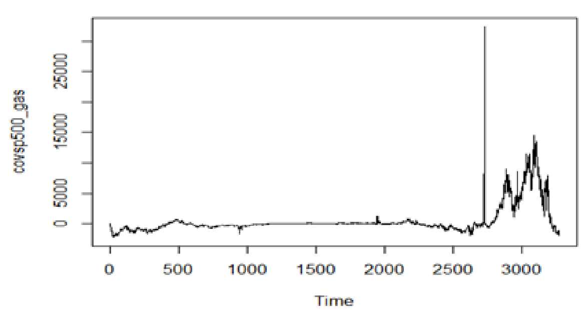
Finally, the diagonal elements of the covariance matrix represent the variances of the individual variables. These variances give an idea of the uncertainty or risk associated with each variable. In this case, the variances are 4,142,109.623 for s&p500, 490.50701 for cued. oil. price, 0.3886354 for natural. gas. price, and 399,632.0163 for gold. price.

Keep in mind that, these covariance values are conditional and time-varying, this mean they can change over time. The values provided here are specific to the last time point in data, representing the most recent relationships between the variables. Always consider the dynamic nature of these relationships when making investment decisions or forecasts.

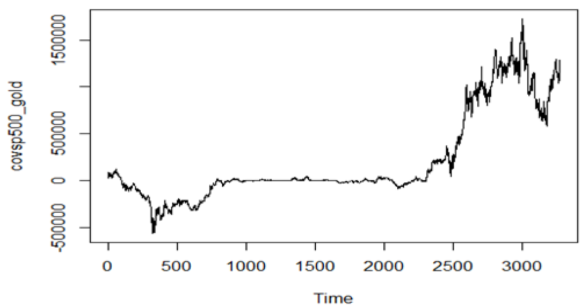
The covariance matrix is an important input for portfolio optimization and risk management, as it helps investors to understand the relationships between asset returns and to construct portfolios that offer the best trade-off between risk and return.



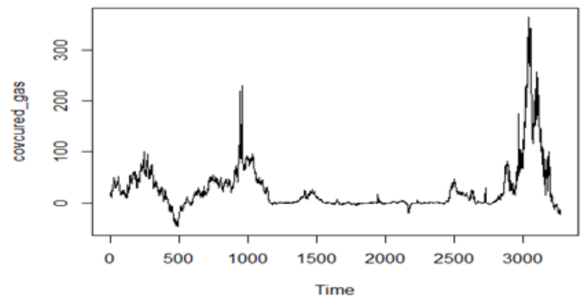
Dc covariance between s&p500 and cured oil price



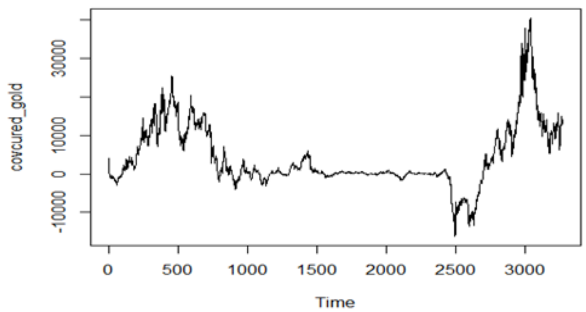
Dc covariance between s&p500 and natural gas price



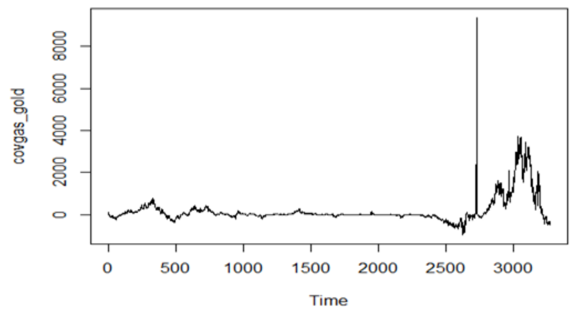
Dc covariance between s&p500 and gold price



Dc covariance between cured oil price and natural gas price



Dc covariance between cured oil price and gold price



Dc covariance between natural gas price and gold price

Figure [3] :DC Covariance plot

The estimated covariances between the assets as shown in figure[3], also exhibit time-varying behavior, further emphasizing the need for a dynamic approach to portfolio management. The fluctuations in covariances can have significant implications for portfolio risk assessment and the potential for risk reduction through diversification.

(5-5) Forecasting

The DCC GARCH forecast provides the predicted dynamic conditional correlations for the assets over a horizon of 365 days. Here, the first two and the last two forecasted correlation matrices are displayed as shown in tables (5) and (6):

Table(5): First 2 Correlation forecasts:

		S&P 500	Crude Oil Price	Natural Gas Price	Gold Price
1st Correlation Forecast	**S&P 500 [t+1]**	**1.0000**	**0.9959**	**-0.9903**	**0.9979**
	Crude Oil Price [t+1]	**0.9959**	**1.0000**	**-0.9865**	**0.9961**
	Natural Gas Price [t+1]	**-0.9903**	**-0.9865**	**1.0000**	**-0.9878**
	Gold Price[t+1]	**0.9979**	**0.9961**	**-0.9878**	**1.0000**
2nd Correlation Forecast	**S&P 500 [t+2]**	**1.0000**	**0.9954**	**-0.9901**	**0.9975**
	Crude Oil Price [t+2]	**0.9954**	**1.0000**	**-0.9860**	**0.9959**
	Natural Gas Price [t+2]	**-0.9901**	**-0.9860**	**1.0000**	**-0.9874**
	Gold Price[t+2]	**0.9975**	**0.9959**	**-0.9874**	**1.0000**

Table (6): last 2 Correlation forecasts:

		S&P 500	Crude Oil Price	Natural Gas Price	Gold Price
2nd to Last Correlation Forecast	**S&P 500 [t+1]**	**1.0000**	**0.8466**	**-0.8957**	**0.8898**
	Crude Oil Price [t+1]	**0.8466**	**1.0000**	**-0.8260**	**0.9209**
	Natural Gas Price [t+1]	**-0.8957**	**-0.8260**	**1.0000**	**-0.8717**
	Gold Price[t+1]	**0.8898**	**0.9209**	**-0.8717**	**1.0000**
last Correlation Forecast	**S&P 500 [t+2]**	**1.0000**	**0.8462**	**-0.8955**	**0.9975**
	Crude Oil Price [t+2]	**0.8462**	**1.0000**	**-0.8256**	**0.9208**
	Natural Gas Price [t+2]	**-0.8955**	**-0.8256**	**1.0000**	**-0.8714**
	Gold Price[t+2]	**0.9975**	**0.9208**	**-0.8714**	**1.0000**

The forecasted correlation matrices show the expected changes in the relationships between the assets over the next 365 days. As we can see, the correlations in the first two forecasts are quite similar to the correlations at the last observation. However, the correlations in the last two forecasts have changed, suggesting that the relationships between the assets may evolve over time.

Investors can use these forecasts to anticipate potential changes in the relationships between the assets and to adjust their portfolios accordingly to achieve optimal diversification and risk management. Note that these forecasts are subject to uncertainty, and the actual correlations may differ from the predicted values.

6- Conclusion:

This study has employed the DCC GARCH model to investigate the dynamic correlations among the S&P 500, Crude Oil Price, Natural Gas Price, and Gold Price. The results reveal significant time-varying correlations among these asset classes, highlighting the importance of considering the evolving nature of interdependencies when making asset allocation decisions.. Further research could explore the impact of macroeconomic factors on dynamic correlations, the application of the DCC-GARCH model to other asset classes, and the development of new methods for portfolio optimization based on time-varying correlations.

Moreover, the results provide a foundation for future research aimed at deepening our understanding of the factors driving asset class correlations and their implications for portfolio management. By employing alternative estimation techniques for the DCC GARCH model, such as the Maximum Likelihood Estimation (MLE) or Bayesian Estimation, to assess the sensitivity of the results to the chosen estimation method.. Also, incorporating tests for structural change and accounting for any identified breaks in the analysis can improve the accuracy of the estimated correlations and forecasting performance.

Incorporating these statistical recommendations in future research will contribute to a more robust analysis of asset class correlations and facilitate the development of more informed and effective asset allocation strategies in the ever-changing financial landscape.

References

1- Alexander, C. (2008). Market Risk Analysis, Volume IV: Value-at-Risk Models. John Wiley & Sons.

2- Ang, A., & Bekaert, G. (2002). International asset allocation with regime shifts. The Review of Financial Studies, 15(4), 1137-1187.

3- Bauwens, L., Laurent, S., &Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. Journal of Applied Econometrics, 21(1), 79-109.

4- Bekaert, G., Hodrick, R. J., & Zhang, X. (2009). International stock return comovements. The Journal of Finance, 64(6), 2591-2626.

5- Billio, M., Getmansky, M., Lo, A. W., &Pelizzon, L. (2013). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics, 104(3), 535-559.

6- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

7- Bollerslev, T. (1990). Modelling the coherence in short-runnominalexchange rates: A multivariate generalized ARCH model. Review of Economics and Statistics, 72(3), 498-505.

8- Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. The Review of Economics and Statistics, 72(3), 498-505.

9- Cappiello, L., Engle, R. F., & Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial Econometrics, 4(4), 537-572.

10-Chiu, E., Wu, H., & Yu, J. (2010). Dynamic correlation and portfolio optimization: An empirical study on financial market uncertainty. Mathematics and Computers in Simulation, 80(10), 2019-2031.

11-Christoffersen, P., Errunza, V., Jacobs, K., & Langlois, H. (2011). Is the potential for international diversification disappearing? A dynamic copula approach. Review of Financial Studies, 24(12), 3711-3751.

12-Chua, C. L., Suardi, S., &Tsiaplias, S. (2017). A multivariate GARCH model incorporating the direct and indirect transmission of conditional variances and the cross-market effect. Journal of Time Series Econometrics, 9(1), 59-77.

13-Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business & Economic Statistics, 20(3), 339-350.

14-Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50(4), 987-1007.

15-Huang, D., Yu, H., & Yu, J. (2009). Value-at-risk analysis for Taiwan stock index futures: Fat tails and conditional asymmetries in return innovations. Review of Pacific Basin Financial Markets and Policies, 12(1), 1-23.

16-Huang, Y., Su, W., & Li, X. (2010). Comparison of BEKK GARCH and DCC GARCH models: An empirical study. In Advanced Data Mining and Applications: 6th International Conference, ADMA 2010, Chongqing, China, November 19-21, 2010, Proceedings, Part II 6 (pp. 99-110). Springer Berlin Heidelberg.

17-Kose, M. A., Otrok, C., & Whiteman, C. H. (2006). Understanding the evolution of world business cycles. Journal of International Economics, 70(1), 110-130.

18-Kroner, K. F., & Ng, V. K. (1998). Modeling asymmetric comovements of asset returns. The Review of Financial Studies, 11(4), 817-844.

19-Longin, F., &Solnik, B. (2001). Extreme correlation of international equity markets. The Journal of Finance, 56(2), 649-676.

20-Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1), 77-91.

21-Silvennoinen, A., &Teräsvirta, T. (2009). Multivariate GARCH models. In Handbook of Financial Time Series (pp. 201-229). Springer, Berlin, Heidelberg.

22-Silvennoinen, A., &Teräsvirta, T. (2009). Multivariate GARCH models. In Handbook of Financial Time Series (pp. 201-229). Springer, Berlin, Heidelberg.

23-Sheedy, E. A. (2009). Can risk modeling work?. Macquarie University Applied Finance Centre Research Paper, (35).

24-Tse, Y. K., & Tsui, A. K. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. Journal of Business & Economic Statistics, 20(3), 351-362.

25-Teräsvirta, T. (2009). An introduction to univariate GARCH models. In Handbook of financial time series (pp. 17-42). Berlin, Heidelberg: Springer Berlin Heidelberg.

26-Zhang, K., & Chan, L. (2009). Efficient factor garch models and factor-dcc models. Quantitative Finance, 9(1), 71-91.

ملخص:

في الأسواق المالية، يعد فهم العلاقات الديناميكية بين الأصول أمراً بالغ الأهمية لإدارة المحافظ الاستثمارية بشكل فعال. تسلط هذه الدراسة الضوء على أهمية استخدام نموذج DCC-GARCH (الارتباط الديناميكي المشروط - الانحدار الذاتي المشروط المتغاير) كأداة تحليل قوية متعددة المتغيرات لالتقاط الارتباطات الديناميكية بين مؤشر S&P 500 وسعر النفط الخام وسعر الغاز الطبيعي وسعر الذهب. يوفر نموذج DCC-GARCH إطاراً مرناً لنمذجة الارتباطات المتغيرة بمرور الوقت، مما يسمح للمستثمرين بحساب العلاقات المتغيرة بين الأصول بمرور الوقت. وتقدر الدراسة الارتباطات وتتنبأ بتطورها خلال الـ 365 يومًا القادمة، مما يوفر رؤى قيمة لتحسين المحفظة وإدارة المخاطر. النتائج وتوضح النتائج فوائد التنويع المحتملة التي توفرها هذه الأصول وتؤكد على الحاجة إلى إدارة المحفظة التكيفية على أساس الارتباطات الديناميكية. ومن خلال استخدام نموذج DCC-GARCH، يمكن للمستثمرين فهم التفاعلات المعقدة بين الأصول بشكل أفضل واتخاذ قرارات أكثر استنارة بشأن تخصيص الأصول، مما يؤدي في النهاية إلى تحسين العوائد المعدلة حسب المخاطر. تؤكد هذه الدراسة على أهمية دمج التقنيات المتقدمة متعددة المتغيرات، مثل DCC-GARCH، في التحليل المالي وإدارة المحافظ الاستثمارية.

الكلمات المفتاحية: الارتباطات الديناميكية، DCC-GARCH، التحليل متعدد المتغيرات، الارتباطات المتغيرة بمرور الوقت، إدارة المحافظ، الأسواق المالية.