Statistical Inference for Two Burr Type XII Populations Based on Joint Progressive Type II Censored Scheme

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Abstract

Recently the joint progressive type II censoring scheme is useful for planning comparative purposes of two identical products manufactured coming from different lines. In this paper, we consider the life time Burr type XII distribution with jointly progressive type-II censoring scheme. The maximum likelihood estimators of the parameters and Bayes estimators have been developed using Markov chain Monte Carlo by utilizing Metropolis-Hasting algorithm under squared error and linear-exponential loss functions. In Bayesian approach the Markov chain Monte Carlo method is adopted to compute estimates. Moreover, we obtain both approximate and Highest posterior density credible intervals. Monte Carlo results from simulation studies have been presented to assess the performance of our proposed methods. Finally a real data set has been analyzed for illustrative purposes.

Keywords: Burr type XIIdistribution; Joint progressive type-II censoring; Maximum likelihood estimation; Confidence intervals; Bayesian estimation; Loss function; Markov chain Monte Carlo.

1. Introduction

The two parameter Burr type XII distribution (denoted by Burr XII distribution) was introduced as a member of the Burr (1942) family of distributions which includes 12 types of cumulative distribution functions with a variety of density shapes. Among those 12 distributions functions, Burr Type XII distribution has received the most attention in the statistical literature. This distribution plays major role in the analyses of lifetime and survival data. Due to its flexibility and some desirable properties, applications have proved to be much wide. Applications may be found in areas of quality control, economics, duration of failure time modeling, insurance risk and reliability analysis.

A random variable *X* is said to have Burr XII$(\Re,\aleph)$ distribution, if its probability density function is given by

$$f(x) = \Re\aleph x^{\Re-1}\left(1+x^{\Re}\right)^{-(\aleph+1)}, \;\; x>0, \Re, \aleph > 0$$

and a cumulative distribution function

$$F(x) = 1-\left(1+x^{\Re}\right)^{-\aleph}, \;\; x>0 \; (1)$$

where $\Re$ and $\aleph$ are the shape parameters of the distribution. Statistical inference based on Burr XII$(\Re,\aleph)$ distribution as a lifetime model has been discussed by several authors, see for example, Papadopoulos (1978), Al-Hussaini and Jaheen (1992), Ali Mousa and Jaheen (2002), Jaheen (2005) , Soliman et al. (2013), Jang et al. (2014), Gunasekera (2018) , Panahi (2019), Ateya et al. (2020),Parviz and Panahi (2020) and Yan et al. (2021).

Censoring schemes are used to reduce the costs of experiments and to accelerate design performance. There are various types of censored data to be dealt with in the analysis of lifetime experiments (see Lawless(2003)). Almost all of these types of data are concerned with the one-sample problems. However, there are situations in which the experimenter plans to compare different populations. In such problems, the joint censoring scheme is scheme is quite useful in while conducting comparative life tests of products from different units within the same facility. More clearly for joint censoring scheme, suppose that products are being produced by two different lines under the same facility, and that two independent samples of sizes m and n are selected from these lines and placed simultaneously on a life-testing experiment. In order to save time and money, suppose the experimenter chooses to terminate the life-testing experiment when a certain number of failures occur (say, r). Under joint Type-II censoring, specimens of two products under study are placed on a life-test simultaneously, successive failure times and the corresponding product types will be recorded, and the life-testing experiment will get terminated as soon as a pre-specified number of failures (say, r) are observed. Balakrishnan and Rasouli (2008) studied the exact likelihood inference for two exponential populations under joint Type-II censoring. If an experimenter desires to remove live units at points other than the termination point of the life test, the above described scheme will not be of use to the experimenter. The joint Type-II censoring does not allow for units to be lost or removed from the test at points other than the final termination point. So, more general censoring schemes are required.

Rasouli and Balakrishnan (2010) introduced joint progressive type-II censoring (JPC-II) as follows:

Suppose $X_1,...,X_m$ the lifetimes of m specimens of product 1, and are (iid) random variables from a population with distribution function $F_1(x)$ and density function $f_1(x)$, and $Y_1,...,Y_n$ the lifetimes of n specimens of product 2 , and are (iid) random variables from a population with distribution function $F_2(x)$ and density function $f_2(x)$. All $N = m + n$ items are put to life testing at time zero and the experiment is terminated as soon as r failures, either from product 1 or from product 2, are observed. To run the experiment according to a joint progressive Type II censoring scheme, the following algorithm is used:

(1) At the time of the first failure (that may be from either X or Y), R_1 units are randomly withdrawn from the remaining $N-1$ surviving units.

(2) Similarly, at the time of the second failure (which may be from either X or Y), R_2 units are randomly withdrawn from the remaining $N - R_1 - 2$ surviving units and so on.

(3) Finally, When the r^{th} failure is observed, all the remaining $R_r = N - r - R_1 - R_2 - ... - R_{r-1}$ surviving units are withdrawn from the life-testing experiment.

Here, The progressive type-II censoring scheme $R = (R_1, R_2,..., R_r)$ has the decomposition $S + Q = (s_1,...,s_r) + (q_1,...,q_r)$, where $R = S + Q$, $S(Q)$ is the number of units withdrawn at the time of the i^{th} failure that belongs to X (Y) sample and these are unknown and random variables. Thus, The available data consist of (ϑ, R, W) where $W=\left(w_{(1)},\dots,w_{(r)}\right)$ with $r<N$ being a prefixed integer, $\vartheta=(\vartheta_1,\dots,\vartheta_r)$ with $\vartheta_i=1$ or *0* if $w_{(i)}$ is from $X-orY-$failure respectively. The likelihood of (ϑ, W, and S) is given by

$$L=c_{\mathrm{K}}\prod_{i=1}^{r}\left(f_1\left(w_{(i)}\right)\right)^{\vartheta_i}\left(f_2\left(w_{(i)}\right)\right)^{1-\vartheta_i}\left(\bar{F}_1\left(w_{(i)}\right)\right)^{s_i}\left(\bar{F}_2\left(w_{(i)}\right)\right)^{q_i},\quad(2)$$

Where $\bar{F}=1-F$, $\sum_{i=1}^{r}s_i+\sum_{i=1}^{r}q_i=\sum_{i=1}^{r}R_i$, $\sum_{i=1}^{r}s_i=m-m_r$, $\sum_{i=1}^{r}q_i=n-n_r$ and $c_{\mathrm{K}}=D_1D_2$, such that

$$D_1=\prod_{j=1}^{r}\left(\left(m-\sum_{i=1}^{j-1}\vartheta_i-\sum_{i=1}^{j-1}s_i\right)\vartheta_j+\left(n-\sum_{i=1}^{j-1}(1-\vartheta_i)-\sum_{i=1}^{j-1}(R_i-s_i)\right)\left(1-\vartheta_j\right)\right),$$

and

$$D_2=\prod_{j=1}^{r-1}\left(\frac{\binom{m-\sum_{i=1}^{j-1}\vartheta_i-\sum_{i=1}^{j-1}s_i}{s_j}\binom{n-\sum_{i=1}^{j-1}(1-\vartheta_i)-\sum_{i=1}^{j-1}(R_i-s_i)}{q_j}}{\binom{m+n-j-\sum_{i=1}^{j-1}R_i}{R_j}}\right)$$

Several authors have addressed inferential issues based on JPC-II samples; for example: Rasouli and Balakrishnan (2010) discussed exact likelihood inference for the parameters of two exponential populations when JPC-II is implemented on the two samples. They developed exact inferential methods based on maximum likelihood estimators (MLEs) and compared their performance with those based on approximate, Bayesian and bootstrap methods, under JPC-II scheme assuming exponential for both samples. Parsi et al. (2011) developed inference of the parameters of two Weibull populations under JPC-II, presented the details of the proposed model and derives the MLEs of the model parameters. Doostparast et al. (2013) considered the Bayesian inference for the unknown parameters of two Weibull populations under JPC-II by using squared error(SE) and linear-exponential(LINEX) loss function. Torabi et.al (2015) discussed general JPC-II censoring scheme and inference for parameters of two weibull populations under this scheme. They obtained the MLEs and confidence interval using procedures such as asymptotic normality and bootstrap methods, under the scheme. Finally, by means a simulation study these estimations are evaluated and also all confidence intervals are compared in terms of coverage probabilities. Abo-Kasem (2020) discussed statistical inferences for two Rayleigh populations based on JPC-II censoring scheme. Heobtained the MLEs of the unknown parameters when it exists, Bayes estimators for the unknown parameters using SE and LINEX loss functions and both approximate and Bayes credible confidence intervals. The theoretical results of point and interval estimation obtained are assessment and compared through illustrative example and simulation studies. Mondal and Kundu (2020) considered the JPC-II scheme for two populations when the lifetime distributions of the experimental units of the two populations follow two-parameter generalized exponential distributions with the same scale parameter but different shape parameters. Krishna and Goal (2020) dealed with inferences for Lindley populations, when JPC-II censoring scheme is applied on two samples in a joint manner. They obtained the MLEs of parameters along with their associated confidence intervals which dependent on Fisher's information matrix and Bayes estimators of parameters are considered. A Monte Carlo simulation study is performed to measure the efficiency of the estimates also a real data set is given for illustrative purpose. Aljohani (2021) discussed statistical inference of Chen Distribution populations under JPC-II censoring. He obtained the MLEs and Bayes estimators of the unknown parameters. The theoretical results are obtained through simulation studies and verified in an analysis of the lifetime data.

The rest of this paper is organized as follows. In Section 2, the MLEs and asymptotic confidence intervals are obtained. In Section 3, the Bayes estimators under squared error (SE) and linear-exponential (LINEX) loss functions and HPD intervals for the parameters using JPC-II scheme are derived. In Section 4, the theoretical results of point and interval estimation compared through illustrative example and simulation studies are given. In Section 5, a real data analysis is presented. Finally conclusion is given in Section 6.

2.Maximum Likelihood Estimation

Let the two populations are Burr type XII distribution with equation (1).In this case, the likelihood function in (2) becomes

$$L = c_{\mathrm{K}} \prod_{i=1}^{r} \left[\left(\Re_1 \aleph_1 w_{(i)}^{\Re_1 - 1} \left(1 + w_{(i)}^{\Re_1}\right)^{-(\aleph_1 + 1)} \right)^{\vartheta_i} \left(\Re_2 \aleph_2 w_{(i)}^{\Re_2 - 1} \left(1 + w_{(i)}^{\Re_2}\right)^{-(\aleph_2 + 1)} \right)^{1-\vartheta_i} \right]$$

$$\left(\left(1 + w_{(i)}^{\Re_1}\right)^{-\aleph_1} \right)^{s_i} \left(\left(1 + w_{(i)}^{\Re_2}\right)^{-\aleph_2} \right)^{q_i} \quad (3)$$

Taking natural logarithm of L gives:

$$
\begin{aligned}
\ln L = {} & \ln\left(c_{K}\Re_1^{m_r}\aleph_1^{m_r}\Re_2^{n_r}\aleph_2^{n_r}\right)+\left(\Re_1-1\right)\sum_{i=1}^{r}\vartheta_i \ln w_{(i)}-\left(\aleph_1+1\right)\sum_{i=1}^{r}\vartheta_i \ln\left(1+w_{(i)}^{\Re_1}\right) \\
& +\left(\Re_2-1\right)\sum_{i=1}^{r}\left(1-\vartheta_i\right)\ln w_{(i)}-\left(\aleph_2+1\right)\sum_{i=1}^{r}\left(1-\vartheta_i\right)\ln\left(1+w_{(i)}^{\Re_2}\right) \\
& -\aleph_1\sum_{i=1}^{r}s_i \ln\left(1+w_{(i)}^{\Re_1}\right)-\aleph_2\sum_{i=1}^{r}q_i \ln\left(1+w_{(i)}^{\Re_2}\right) \quad (4)
\end{aligned}
$$

The point estimation for $\Re_h$ and $\aleph_h$ $(h=1,2)$ can be obtained by finding the first derivatives of the natural logarithm of the likelihood function (4) with respect to $\Re_h$ and $\aleph_h$ and equating the new equations to zero, so we get the following equations

$$
\frac{m_r}{\hat{\Re}_1}+\sum_{i=1}^{r}\vartheta_i \ln w_{(i)}-\left(\hat{\aleph}_1+1\right)\sum_{i=1}^{r}\vartheta_i \frac{w_{(i)}^{\hat{\Re}_1}\ln w_{(i)}}{\left(1+w_{(i)}^{\hat{\Re}_1}\right)}-\hat{\aleph}_1\sum_{i=1}^{r}s_i \frac{w_{(i)}^{\hat{\Re}_1}\ln w_{(i)}}{\left(1+w_{(i)}^{\hat{\Re}_1}\right)}=0,
$$

$$
\frac{n_r}{\hat{\Re}_2}+\sum_{i=1}^{r}\left(1-\vartheta_i\right)\ln w_{(i)}-\left(\hat{\aleph}_2+1\right)\sum_{i=1}^{r}\left(1-\vartheta_i\right)\frac{w_{(i)}^{\hat{\Re}_2}\ln w_{(i)}}{\left(1+w_{(i)}^{\hat{\Re}_2}\right)}-\hat{\aleph}_2\sum_{i=1}^{r}q_i \frac{w_{(i)}^{\hat{\Re}_2}\ln w_{(i)}}{\left(1+w_{(i)}^{\hat{\Re}_2}\right)}=0,
$$

$$
\frac{m_r}{\hat{\aleph}_1}-\sum_{i=1}^{r}\vartheta_i \ln\left(1+w_{(i)}^{\hat{\Re}_1}\right)-\sum_{i=1}^{r}s_i \ln\left(1+w_{(i)}^{\hat{\Re}_1}\right)=0,
$$

and

$$
\frac{n_r}{\hat{\aleph}_2}-\sum_{i=1}^{r}\left(1-\vartheta_i\right)\ln\left(1+w_{(i)}^{\hat{\Re}_2}\right)-\sum_{i=1}^{r}q_i \ln\left(1+w_{(i)}^{\hat{\Re}_2}\right)=0 \quad (5)
$$

By solving equations (5) we obtain the MLEs of the parameters $\Re_1,\aleph_1,\Re_2$ and $\aleph_2$, it’s clear that, the analytical solution may be very difficult to find. So, we use a numerical methods to obtain $\hat{\Re}_1,\hat{\aleph}_1,\hat{\Re}_2$ and $\hat{\aleph}_2$

The asymptotic variance-covariance matrix for $\Re_1,\aleph_1,\Re_2$ and $\aleph_2$ is obtained by inverting the information matrix through the elements that are negative of the expected values of the second order derivatives of the logarithms of likelihood functions. The elements of the sample information matrix will be

$$-\frac{\partial^2 \ln L}{\partial \Re_1^2}=\frac{m_r}{\Re_1^2}+\left(\aleph_1+1\right)\sum_{i=1}^{r}\vartheta_i\,\frac{w_{(i)}^{\Re_1}\ln w_{(i)}+\left(w_{(i)}^{\Re_1}\right)^2\ln w_{(i)}-\left(w_{(i)}^{\Re_1}\right)^2\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_1}\right)^2}$$

$$+\aleph_1\sum_{i=1}^{r}s_i\,\frac{w_{(i)}^{\Re_1}\ln w_{(i)}+\left(w_{(i)}^{\Re_1}\right)^2\ln w_{(i)}-\left(w_{(i)}^{\Re_1}\right)^2\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_1}\right)^2},$$

$$-\frac{\partial^2 \ln L}{\partial \Re_2^2}=\frac{n_r}{\Re_2^2}+\left(\aleph_2+1\right)\sum_{i=1}^{r}\left(1-\vartheta_i\right)\frac{w_{(i)}^{\Re_2}\ln w_{(i)}+\left(w_{(i)}^{\Re_2}\right)^2\ln w_{(i)}-\left(w_{(i)}^{\Re_2}\right)^2\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_2}\right)^2}$$

$$+\aleph_2\sum_{i=1}^{r}q_i\,\frac{w_{(i)}^{\Re_2}\ln w_{(i)}+\left(w_{(i)}^{\Re_2}\right)^2\ln w_{(i)}-\left(w_{(i)}^{\Re_2}\right)^2\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_2}\right)^2},$$

$$-\frac{\partial^2 \ln L}{\partial \aleph_1^2}=\frac{m_r}{\aleph_1^2},$$

$$-\frac{\partial^2 \ln L}{\partial \aleph_2^2}=\frac{n_r}{\aleph_2^2},$$

$$-\frac{\partial^2 \ln L}{\partial \Re_1 \partial \aleph_1}=\sum_{i=1}^{r}\vartheta_i\,\frac{w_{(i)}^{\Re_1}\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_1}\right)}+\sum_{i=1}^{r}s_i\,\frac{w_{(i)}^{\Re_1}\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_1}\right)},$$

and

$$-\frac{\partial^2 \ln L}{\partial \Re_2 \partial \aleph_2}=\sum_{i=1}^{r}\left(1-\vartheta_i\right)\frac{w_{(i)}^{\Re_2}\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_2}\right)}+\sum_{i=1}^{r}q_i\,\frac{w_{(i)}^{\Re_2}\ln w_{(i)}}{\left(1+w_{(i)}^{\Re_2}\right)}\,(6)$$

Suppose that $\hat{\delta}$ is the MLE of the parameter vector $\delta = (\Re_1, \aleph_1, \Re_2, \aleph_2)$ .Under some regularity conditions, $\hat{\delta}$ is approximately normal with mean δ and covariance matrix I_{δ}^{-1}. Practically, we estimate I_{δ}^{-1} by $I_{\hat{\delta}}^{-1}$, then

$$I_{\hat{\delta}}^{-1} \cong \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \Re_1^2} & 0 & -\frac{\partial^2 \ln L}{\partial \Re_1 \partial \aleph_1} & 0 \\ 0 & -\frac{\partial^2 \ln L}{\partial \Re_2^2} & 0 & -\frac{\partial^2 \ln L}{\partial \Re_2 \partial \aleph_2} \\ -\frac{\partial^2 \ln L}{\partial \Re_1 \partial \aleph_1} & 0 & -\frac{\partial^2 \ln L}{\partial \aleph_1^2} & 0 \\ 0 & -\frac{\partial^2 \ln L}{\partial \Re_2 \partial \aleph_2} & 0 & -\frac{\partial^2 \ln L}{\partial \aleph_2^2} \end{bmatrix}_{\hat{\delta}}^{-1}$$

$$\cong \begin{bmatrix} \mathrm{var}(\hat{\Re}_1) & 0 & \mathrm{cov}(\hat{\Re}_1, \hat{\aleph}_1) & 0 \\ 0 & \mathrm{var}(\hat{\Re}_2) & 0 & \mathrm{cov}(\hat{\Re}_2, \hat{\aleph}_2) \\ \mathrm{cov}(\hat{\Re}_1, \hat{\aleph}_1) & 0 & \mathrm{var}(\hat{\aleph}_1) & 0 \\ 0 & \mathrm{cov}(\hat{\Re}_2, \hat{\aleph}_2) & 0 & \mathrm{var}(\hat{\aleph}_2) \end{bmatrix}$$

Now, the approximate confidence intervals of $\Re_h$ and $\aleph_h$, $h = 1,2$ with confidence level $100(1-\alpha)\%$ are given by

$$\hat{\Re}_h \pm z_{(1-\alpha/2)} \sqrt{\mathrm{var}(\hat{\Re}_h)} \text{ and } \hat{\aleph}_h \pm z_{(1-\alpha/2)} \sqrt{\mathrm{var}(\hat{\aleph}_h)}, h = 1,2 .$$

Where $z_{(1-\alpha/2)}$ denotes the upper $(1-\alpha/2)$ percentage point of the standard normal distribution.

3. Bayesian Estimation

In this section, the Bayes estimators using SE and LINEX loss functions under the assumption of gamma prior for the unknown parameters $\Re_h$ and $\aleph_h$ will be obtained. We consider that $\Re_1, \aleph_1, \Re_2$ and $\aleph_2$ have the following independent gamma prior distributions;

$$\pi\left(\Re_h\right) \propto \frac{a_h^{b_h}}{\Gamma\left(b_h\right)} \Re_h^{b_h-1} e^{-a_h \Re_h} \ , a_h, b_h, \Re_h > 0,$$

and

$$\pi\left(\aleph_h\right) \propto \frac{a_h^{b_h}}{\Gamma\left(b_h\right)} \aleph_h^{b_h-1} e^{-a_h \aleph_h}, a_h, b_h, \aleph_h > 0 \ , h = 1,2 \ (7)$$

Here all the hyper parameters a_h and b_h are assumed to be known and non-negative.Combining (7) with equation (3) and using Bayes theorem, the joint posterior density function of $\Re_1, \aleph_1, \Re_2$ and $\aleph_2$ can be written as:

$$\pi\left(\Re_1, \aleph_1, \Re_2, \aleph_2 \middle| x\right) \propto \frac{1}{\psi} L \ \pi\left(\Re_h\right) \pi\left(\aleph_h\right) (8)$$

Where $\psi = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty L \ \pi\left(\Re_h\right) \pi\left(\aleph_h\right) d\Re_h d\aleph_h \ , h = 1,2$.

Therefore, the Bayes estimator of any function of $\Re_1, \aleph_1, \Re_2$ and $\aleph_2$, say $\delta\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right)$ under the SE loss function is

$$\tilde{\delta} = E_{\Re_1, \aleph_1, \Re_2, \aleph_2 / x} \left(\delta\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right)\right)$$

$$= \frac{1}{\psi} \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \delta\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right) L \ \pi\left(\Re_h\right) \pi\left(\aleph_h\right) d\Re_h d\aleph_h \ (9)$$

Under a LINEX loss function the Bayes estimate of a function $\delta\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right)$ is given by

$$\delta^* = -\frac{1}{\tau} \ln E\left(e^{-\tau\delta}\right), \quad \tau \neq 0, (10)$$

where $E\left(e^{-\tau\delta}\right)=\dfrac{1}{\psi}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\tau\delta}L\ \pi\left(\Re_h\right)\pi\left(\aleph_h\right)d\Re_h d\aleph_h\ , h=1,2$.

Equations (6), (7), (9) and (10) are hard to obtain, so Markov chain Monte Carlo (MCMC) approach can be suggested as an approximation of the Bayes estimates of $\Re_1,\aleph_1,\Re_2,\aleph_2$ and generating a posterior sampling using Metropolis-Hasting(MH) algorithm.

Metropolis-Hasting Algorithm

Suppose our goal is to draw samples from the posterior density (8), therefore the MH generates a sequence of draws. To perform the MH algorithm for Burr type XII distribution, we have to start with simulating a candidate sample δ' from the proposal distribution □ (δ). Samples from the proposal distribution are not accepted automatically as posterior samples, these candidate samples are accepted probabilistically based on the acceptance probability. more clearly for the steps of MH algorithm to draw a sample, follow the following steps:

Step 1.Set $i=1$.

Step 2. Start with any initial value $\delta^{(i-1)}$.

Step 3. Using the initial value, sample a candidate point δ' from proposal

distribution □ (δ).

Step 4. Given the candidate point δ', Calculate the acceptance probability

$$B=\min\left(1,\frac{\pi\left(\delta'|x\right)}{\pi\left(\delta^{(i-1)}|x\right)}\right)$$

where $\pi(.)$ is the posterior density in (8).

Step 5. Draw a value of u from the uniform distribution $U(0,1)$.

Step 6. Accept or reject the new candidate δ'

$$\begin{cases} \text{If } u \le B \quad set \quad \delta^{(i)} = \delta' \\ \text{otherwise} \quad \text{set} \quad \delta^{(i)} = \delta^{(i-1)} \end{cases}$$

Step 7.Set $i = i + 1$, and repeat steps 2-7 M times until we get M draws.

Finally, from the random samples of size M drawn from the posterior density, some of the initial samples can be discarded (burn-in), and remaining samples can be further carried out to calculate Bayes estimates. More accurately (9) can be estimated as

$$\tilde{\delta} = \frac{1}{M - l_b} \sum_{i=l_b}^{M} \delta^{(i)}$$

where M is the sample size drawn from the posterior density and l_b represent the number of burn-in samples (Dey and Pradhan (2014)).

Highest Posterior Density Intervals

The technique of Chen and Shao (1999) has been broadly utilized for constructing highest posterior density (HPD) intervals for δ of Burr type XII distribution under JPC-II. In this sub-section, the samples drawn using the proposed MH algorithm shall be employed to construct the interval estimates. More accurately, let us assume that $\Pi(\delta|x)$ denotes the posterior distribution function of δ. Let us further suppose that $\delta^{(\alpha)}$ be the α^{th} quantile of δ, that is,

$$\delta^{(\alpha)} = \inf\left(\delta : \prod\left(\delta | x\right) \geq \alpha\right)$$

where $0 < \alpha < 1, \inf$ is meaning infinimum. Notice that for a given $\delta^{\bullet}$, a simulation consistent estimator of $\prod\left(\delta^{\bullet} | x\right)$ can be estimated as

$$\prod\left(\delta^{\bullet} | x\right) = \frac{1}{M - l_b} \sum_{i=l_b}^{M} I_{\delta \leq \delta^{\bullet}}$$

Where M is the sample size drawn from the posterior density, l_b represent the number of burn-in samples and $I_{\delta \leq \delta^{\bullet}}$ is the indicator function defined as

$$I_{\delta \leq \delta^{\bullet}} = \begin{pmatrix} 1 & if & \delta < \delta^{\bullet} \\ 0 & & otherwise \end{pmatrix}$$

Then the corresponding estimate is obtained as

$$\hat{\prod}\left(\delta^{\bullet} | x\right) = \begin{cases} 0 & if & \delta^{\bullet} < \delta_{(l_b)} \\ \sum_{j=l_b}^{i} \omega_j & if & \delta_{(i)} < \delta^{\bullet} < \delta_{(i+1)} \\ 1 & if & \delta^{\bullet} < \delta_{(M)} \end{cases}$$

Where $\omega_j = \frac{1}{M - l_b}$ and $\delta_{(j)}$ are the ordered values of δ_j. Now, for $i = l_b, ..., M$, $\delta^{(\alpha)}$ can be approximated by

$$\tilde{\delta}^{(\alpha)} = \begin{cases} \delta_{(l_b)}\, if & \alpha = 0 \\ \delta_{(i)}\, if & \sum_{j=l_b}^{i-1} \omega_j < \alpha < \sum_{j=l_b}^{i} \omega_j \end{cases}$$

Now, a $100(1-\alpha)\%$ HPD credible interval for δ ,let

$$HPD_j = \left(\tilde{\delta}^{\left(\frac{j}{M}\right)}, \tilde{\delta}^{\left(\frac{j+(1-\alpha)M}{M}\right)} \right)$$

for $j = l_b, \ldots, (\alpha M)$. Then choose HPD_j. among all the $HPD_j's$ such that it has the smallest width (see Chen and Shao (1999)).

4. Simulation results

The simulation study is conducted by considering different values of sample sizes for the two populations as m = 30,50,60 and n = 30,50,60 , different choices of joint progressive type-II censoring schemes with r = 24,36,48,40,60,80,48,72,96 for example, and by choosing $(\Re_1, \aleph_1, \Re_2, \aleph_2) = (1.5, 0.5, 2, 0.75)$. For all these cases, the MLEs, root mean squared errors $\sqrt{MSE}$ and the 95% simultaneous confidence intervals for $(\Re_1, \aleph_1, \Re_2, \aleph_2)$ and the corresponding coverage probabilities are computed. The Bayesian estimates of $(\Re_1, \aleph_1, \Re_2, \aleph_2)$ under the SE and Linex loss functions are also computed based on 1000 simulations and compute the average values of all the estimates. The average value of the MLEs $\left(\hat{\Re}_1, \hat{\aleph}_1, \hat{\Re}_2, \hat{\aleph}_2\right)$ and $\left(\sqrt{MSE}\right)$ are summarized in Table 1. In Table 2 the coverage probabilities and the average widths of 95% CIs of $(\Re_1, \aleph_1, \Re_2, \aleph_2)$ for approximate confidence intervals are presented for some small, moderate and large values of m , n and r .Bayesian estimates of $(\Re_1, \aleph_1, \Re_2, \aleph_2)$ for different choices of m , n and r are presented in Table 3, and HPD credible intervals of $(\Re_1, \aleph_1, \Re_2, \aleph_2)$ in Table 4.

Table 1 : The average values of the MLEs of $\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right)$ and $\left(\sqrt{MSE}\right)$ for small, moderate and large values of m , n and r

(m,n)	r	Scheme (R)	$\hat{\Re}_1$	$\sqrt{MSE}$	$\hat{\aleph}_1$	$\sqrt{MSE}$	$\hat{\Re}_2$	$\sqrt{MSE}$	$\hat{\aleph}_2$	$\sqrt{MSE}$
(30,30)	24	(4(9),0(15))	1.639	0.928	0.355	0.612	1.879	0.794	0.132	1.627
		(0(15),4(9))	2.249	1.168	1.514	1.170	2.323	0.676	0.599	0.362
	36	(3(8),0(28))	1.663	0.563	0.484	0.164	2.161	0.556	0.639	0.184
		(0(28),3(8))	2.080	0.819	1.519	1.136	2.153	0.533	0.483	0.289
	48	(6(2),0(46))	1.682	0.668	0.502	0.152	2.159	0.492	0.755	0.187
		(0(46),6(2))	1.996	0.720	0.928	0.499	1.897	0.427	0.493	0.279
(50,50)	40	(10(6),0(34))	1.734	0.673	0.459	0.164	2.229	0.551	0.631	0.195
		(0(34),10(6))	1.931	0.649	1.289	0.842	2.128	0.439	0.514	0.259
	60	(5(8),0(52)	1.639	0.396	0.486	0.122	2.209	0.437	0.689	0.152
		(0(52),5(8))	1.978	0.384	1.464	1.022	1.899	0.083	0.475	0.082
	80	(1(20),0(60)	1.558	0.295	0.494	0.105	2.072	0.356	0.682	0.130
		(0(60),1(20))	1.766	0.421	0.823	0.365	2.076	0.324	0.567	0.207
(60,60)	48	(9(8),0(40))	1.646	0.412	0.464	0.141	2.201	0.435	0.622	0.189
		(0(40),9(8))	1.929	0.628	1.318	0.864	2.121	0.386	0.522	0.249
	72	(8(6),0(66))	1.636	0.389	0.482	0.114	2.148	0.377	0.722	0.138
		(0(66),8(6))	2.094	0.708	1.599	1.147	1.943	0.288	0.444	0.313
	96	(12(2),0(94))	1.595	0.324	0.496	0.101	2.089	0.318	0.752	0.130
		(0(94),12(2))	1.899	0.254	0.939	0.225	1.797	0.112	0.485	0.075

Table 2 : Simulated coverage probabilities (CP) and the average widths of the 95% confidence intervals for $\left(\Re_1, \aleph_1, \Re_2, \aleph_2\right)$ for some small, moderate and large values of m , n and r

(m,n)	r	Scheme (R)	$\hat{\Re}_1$ Length	$\hat{\Re}_1$ CP%	$\hat{\aleph}_1$ Length	$\hat{\aleph}_1$ CP%	$\hat{\Re}_2$ Length	$\hat{\Re}_2$ CP%	$\hat{\aleph}_2$ Length	$\hat{\aleph}_2$ CP%
(30,30)	24	(4(9),0(15))	3.439	96.38	1.522	100.00	3.077	95.47	3.082	100.00
		(0(15),4(9))	3.517	95.99	2.294	96.09	2.330	95.29	1.245	98.49
	36	(3(8),0(28))	2.115	95.99	0.644	95.69	2.084	95.69	0.577	96.09
		(0(28),3(8))	2.269	96.00	1.973	96.40	2.004	96.10	0.439	96.90
	48	(6(2),0(46))	2.519	98.20	0.6005	96.30	1.827	95.50	0.729	96.10
		(0(46),6(2))	2.051	95.80	1.016	96.20	1.621	96.10	0.416	96.50
(50,50)	40	(10(6),0(34))	2.476	96.30	0.619	96.40	1.968	96.40	0.600	96.10
		(0(34),10(6))	1.902	95.30	1.149	96.50	1.648	96.20	0.418	96.30
	60	(5(8),0(52)	1.454	94.70	0.478	95.70	1.503	96.30	0.541	95.90
		(0(52),5(8))	1.777	96.192	1.413	96.593	1.207	95.99	0.296	97.194
	80	(1(20),0(60)	1.134	95.50	0.411	97.20	1.369	96.30	0.430	96.70
		(0(60),1(20))	1.279	95.60	0.663	96.80	1.235	96.60	0.392	96.60
(60,60)	48	(9(8),0(40))	1.514	96.00	0.543	95.70	1.515	95.70	0.556	96.30
		(0(40),9(8))	1.796	95.70	1.083	96.60	1.438	96.00	0.394	97.20
	72	(8(6),0(66))	1.435	95.50	0.443	96.30	1.359	96.30	0.533	96.30
		(0(66),8(6))	1.509	96.20	1.293	95.80	1.109	95.90	0.259	96.50
	96	(12(2),0(94))	1.217	95.60	0.394	97.20	1.196	95.90	0.512	96.80
		(0(94),12(2))	1.207	96.60	0.708	96.20	1.044	96.00	0.277	97.40

Table 3 : Bayesian estimates of $(\Re_1,\aleph_1,\Re_2,\aleph_2)$ for different choices of m , n and r

(m,n)	r	Scheme (R)	SE Loss				LINEX Loss			
			$\hat{\Re}_1$	$\hat{\aleph}_1$	$\hat{\Re}_2$	$\hat{\aleph}_2$	$\hat{\Re}_1$	$\hat{\aleph}_1$	$\hat{\Re}_2$	$\hat{\aleph}_2$
(30,30)	24	(4(9),0(15))	1.449	0.444	1.701	0.448	1.278	0.425	1.499	0.434
		(0(15),4(9))	1.850	1.228	2.065	0.592	1.627	1.079	1.839	0.566
	36	(3(8),0(28))	1.557	0.501	1.967	0.679	1.405	0.477	1.793	0.639
		(0(28),3(8))	1.886	1.378	1.962	0.504	1.722	1.254	1.812	0.492
	48	(6(2),0(46))	1.557	0.519	1.995	0.787	1.444	0.502	1.857	0.754
		(0(46),6(2))	1.864	0.916	1.758	0.524	1.727	0.869	1.651	0.510
(50,50)	40	(10(6),0(34))	1.594	0.461	2.057	0.653	1.499	0.449	1.928	0.624
		(0(34),10(6))	1.745	1.166	1.975	0.516	1.615	1.086	1.855	0.505
	60	(5(8),0(52)	1.578	0.492	2.086	0.708	1.495	0.481	1.979	0.689
		(0(52),5(8))	1.978	1.464	1.899	0.475	1.869	1.380	1.817	0.469
	80	(1(20),0(60)	1.511	0.505	1.979	0.701	1.444	0.495	1.894	0.686
		(0(60),1(20))	1.712	0.822	1.987	0.584	1.635	0.798	1.907	0.575
(60,60)	48	(9(8),0(40))	1.571	0.466	2.066	0.635	1.489	0.455	1.951	0.617
		(0(40),9(8))	1.774	1.208	1.994	0.523	1.662	1.136	1.891	0.513
	72	(8(6),0(66))	1.582	0.489	2.052	0.736	1.512	0.479	1.964	0.719
		(0(66),8(6))	2.001	1.519	1.863	0.456	1.907	1.443	1.795	0.451
	96	(12(2),0(94))	1.573	0.505	2.013	0.769	1.492	0.496	1.937	0.753
		(0(94),12(2))	1.847	0.933	1.732	0.503	1.779	0.909	1.678	0.497

Table 4 :HPD credible intervals of $(\Re_1,\aleph_1,\Re_2,\aleph_2)$ for different choices of m , n and r

(m,n)	r	Scheme (R)	HPD Interval							
			$\hat{\Re}_1$		$\hat{\aleph}_1$		$\hat{\Re}_2$		$\hat{\aleph}_2$	
			Length	CP%	Length	CP%	Length	CP%	Length	CP%
(30,30)	24	(4(9),0(15))	1.359	96.898	0.571	96.526	1.580	96.484	0.358	96.199
		(0(15),4(9))	2.009	96.076	1.286	96.593	1.629	96.589	0.559	96.092
	36	(3(8),0(28))	1.313	96.053	0.580	96.396	1.409	95.647	0.515	96.891
		(0(28),3(8))	1.614	96.084	1.332	96.894	1.341	96.493	0.395	97.297
	48	(6(2),0(46))	1.275	96.281	0.519	96.10	1.329	95.573	0.616	97.295
		(0(46),6(2))	1.496	95.591	0.877	96.600	1.125	96.997	0.377	96.600
(50,50)	40	(10(6),0(34))	1.431	95.959	0.556	95.996	1.420	96.375	0.519	97.097
		(0(34),10(6))	1.505	95.70	0.858	96.60	1.279	96.40	0.358	95.80
	60	(5(8),0(52)	1.228	96.80	0.459	98.000	1.229	97.10	0.482	97.000
		(0(52),5(8))	1.435	95.591	1.142	97.99	1.017	95.79	0.301	96.794
	80	(1(20),0(60))	0.974	96.90	0.383	97.70	1.111	96.30	0.409	97.80
		(0(60),1(20))	1.122	96.70	0.612	97.70	1.043	96.60	0.369	96.60
(60,60)	48	(9(8),0(40))	1.193	95.696	0.521	96.20	1.236	96.20	0.480	97.50
		(0(40),9(8))	1.452	95.60	0.833	96.40	1.155	96.90	0.327	96.10
	72	(8(6),0(66))	1.185	95.596	0.432	98.60	1.082	96.49	0.478	96.30
		(0(66),8(6))	1.303	96.80	1.097	96.40	0.935	96.70	0.252	96.20
	96	(12(2),0(94))	1.044	96.30	0.364	96.90	1.018	95.60	0.464	96.50
		(0(94),12(2))	0.983	96.20	0.661	97.60	0.830	96.20	0.269	98.80

The results presented in Table 1 show that the bias of the MLEs is affected by the form of JPC-II employed. From the results presented in Tables 1 and 3, it is clear that the estimates based on the SE and LINEX loss functions yield better results than those of the MLEs. In addition, we observe that for larger number of m, n and r, the MLEs and Bayesian estimators yield better results than when m, n and r are small. From the results presented in Tables 2 and 4, we observe that HPD credible intervals are with shorter width than those based on approximate intervals and for both two interval estimates, when r becomes large, the coverage probabilities rarely improve and get close to the nominal value with shorter width of it when sample sizes m and n are large.

5. Real data analysis

A real data set is analyzed for illustrative purpose as well as to assess the statistical performances of the MLEs and Bayes estimators for the Burr XII distribution under different JPC-II schemes.

The following original data set which provided by Wingo (1993) generated from a clinical trial describing a relief time (in hours) for 30 arthritic patients

0.70, 0.58, 0.54, 0.59, 0.71, 0.55, 0.63, 0.84, 0.49, 0.87, 0.73, 0.72, 0.62, 0.82, 0.84, 0.29, 0.51, 0.61, 0.57, 0.29, 0.36, 0.46, 0.68, 0.34, 0.44, 0.75, 0.39, 0.41, 0.46, 0.66

To illustrate the usefulness of the proposed estimators obtained in Sections 2 and 3 with real situations, we divided the data into two samples by randomly sampling $(m = 15)$ observations and considering these observations as the X sample, and the remaining $(n = 15)$ observations are taken as the Y sample, see Table (5).

Table (5): failure times of ..

Data: X
0.36, 0.57, 0.29, 0.36, 0.58, 0.58, 0.72, 0.46, 0.72, 0.68, 0.84, 0.87, 0.63, 0.59, 0.54
Data: Y
0.70, 0.71, 0.55, 0.49, 0.73, 0.62, 0.82, 0.51, 0.61, 0.34, 0.44, 0.75, 0.39, 0.41, 0.66

Then, we fit Burr XII distribution to each sample and report the results in Table (6). We provided the Kolmogorov-Smirnov test statistic values (K-S) and the corresponding p-values, saying that the data fit the Burr XII distribution with the parameters given in Table (6).

Table (6): MLEs and Kolmogorov-Smirnov test results for data

Data	$\hat{\Re}_1$	$\hat{\Re}_2$	$\hat{\aleph}_1$	$\hat{\aleph}_2$	K-S	p-value
X	4.6780	---	8.1718	---	0.1339	0.9507
Y	---	4.9079	----	9.7225	0.1225	0.9573

Form Table (6), the calculated Kolmogorov-Smirnov (K-S) distance between the empirical and the fitted extended for the Burr XII distribution for the first population (X) is 0.1339 and its p-value is 0.9507 where $\hat{\Re}_1 = 4.6780$ and $\hat{\aleph}_1 = 8.1718$, and for the second population (Y) is 0.1225 and its p-value is 0.9573 where $\hat{\Re}_2 = 4.9079$ and $\hat{\aleph}_2 = 9.7225$ which indicate that this distribution can be considered as an adequate model for the given two data set (X and Y).

From the original data, one can generate, e.g., two JPC-II samples with number of stages $r = 12$ and removed items R_j are assumed as:

- **Scheme I:** $R_1 = R_2 = \cdots = R_{11} = 1,\ R_{12} = 7$. This is can be written as: $(1^{*11}, 7)$
- **Scheme II:** $R_1 = 7,\ R_2 = R_3 = \cdots = R_{12} = 1$. This is can be written as: $(7,1^{*11})$

In Table (7), the MLEs of the parameters $\Re_1, \aleph_1, \Re_2$ and $\aleph_2$ have been calculated at proposed schemes JPC-II samples where two population of failures (X and Y) as in the given real data set and follows Burr XII distribution.

Also, Bayes estimates was computed by utilizing the MH algorithm under the Non-informative prior for SE and LINEX loss functions with initial value of $\tau = 2$ and $\tau = -2$.

It is indicated that, while generating samples from the posterior distribution utilizing the MH algorithm, initial values of $\Re_1, \aleph_1, \Re_2$ and $\aleph_2$ are considered as the MLEs of these parameters. Finally, discarded 2000 burn-in samples among the total 10000 samples created from the posterior density, and subsequently obtained Bayes estimates under two error loss functions (SE and LINEX).

Finally in Table (8), associated asymptotic confidence interval estimates and HPD credible interval are computed.

Table (7): MLE, Bayesian, and standard errors for real data set based on joint progressive Type-II censoring under various censoring schemes

Scheme	Parm.	MLE		Bayes SE		Bayes LINEX $\tau = 2$		Bayes LINEX $\tau = -2$	
		Estimate	St.E	Estimate	St.E	Estimate	St.E	Estimate	St.E
I	$\Re_1$	4.3495	1.2973	3.1072	0.0085	2.6485	0.0088	3.0197	0.0057
	$\aleph_1$	24.3194	24.9542	9.2444	0.0895	2.9225	0.1056	7.4112	0.0589
	$\Re_2$	5.5855	1.4906	3.7836	0.0114	3.1468	0.0120	4.4139	0.0094
	$\aleph_2$	22.9024	23.5686	7.2930	0.0923	2.2814	0.1323	12.7756	0.1366
II	$\Re_1$	4.4796	1.2891	3.5848	0.0116	2.8314	0.0121	3.5993	0.0121
	$\aleph_1$	15.3353	13.2796	9.2436	0.1009	2.7270	0.1026	11.1457	0.134
	$\Re_2$	3.8771	1.1596	2.9403	0.0110	2.2367	0.0111	3.3374	0.0124
	$\aleph_2$	3.0513	1.8515	1.5678	0.0109	1.0551	0.0134	3.4366	0.0356

Parm.-Parameter, St.E-Standard error.

Table (8): Associated interval estimates for MLE and HPD credible interval for real data set based on joint progressive Type-II censoring under various censoring schemes

CI	Scheme	CI for $\Re_1$	CI for $\Re_2$	CI for $\aleph_1$	CI for $\aleph_2$
Approximate	I	(3.5454, 5.1536)	(8.8529, 39.7860)	(4.6616, 6.5094)	(8.2947, 37.5101)
	II	(3.6469, 5.1385)	(5.2524, 14.1441)	(4.8685, 6.6870)	(7.8966, 26.0476)
HPD	I	(1.8140, 4.5663)	(1.0509, 26.0569)	(2.2237, 5.8287)	(0.8330, 23.9039)
	II	(1.7363, 5.8197)	(0.7071, 25.7789)	(4.9686, 3.9177)	(3.4988, 3.3774)

Asy CI- Asymptotic confidence interval.

6. Conclusions

The object of this paper is to discuss different estimation problems as MLE and Bayes estimation of unknown parameters for two Burr type XII populations under JPC-II samples. The MLEs of the parameters, corresponding Fisher information matrix and associated asymptotic confidence interval estimates have been derived. Also, Bayes estimates and associated HPD credible interval have been investigated using MH-algorithm under SE and LINEX loss functions. Finally real data set has been analyzed Wingo (1993) and a simulated study has been conducted to compare the performance of the various proposed estimators. From these results, when the sample sizes of two populations m ,n and the total number of failures rare large, the estimators' biases are small and the confidence intervals have desirable coverage probabilities. Also, it is noted that when r increases, the bias of the MLEs becomes negligible and the confidence length decreases. It can be seen that the coverage probabilities based on the HPD credible intervals better than the approximate confidence intervals.

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الملخص:

في الآونة الأخيرة، تعد المراقبة المشتركة المعجلة من النوع الثاني مفيدا في اختبارات الحياة النسبية ولتخطيط أغراض المقارنة لمنتجين متطابقين تم تصنيعهما من خطوط مختلفة داخل نفس المنشأة. يهدف البحث الي استخدام المراقبة المشتركة المعجلة من النوع الثاني Joint Progressive Type-II Censoring لتقدير معالم توزيع بير النوع الثاني عشر لعينتين والتي له تطبيقات عديدة في اختبارات الحياة وذلك من المنظور البيزي ومن المنظور غير البيزي . بالنسبة للتقدير غير البيزي تم اشتقاق تقدير الامكان الأعظم maximum likelihood estimation وفترات الثقة التقاربية. أما بالنسبة للتقدير البيزي فلقد تم حساب مقدرات بيز اعتمادا على دالة الخسارة لمربع الخطأ ودالة الخسارة الأسية-الخطية باستخدام خوارزمية Metropolis-Hasting ، وكذلك حساب فترات ثقة لهذا التقدير ذات مصداقية عالية للكثافة اللاحقة المناظرة highest posterior density. تم مناقشة طريقتي التقدير من خلال النتائج العددية في ضوء بيانات واقعية وباستخدام طريقة المحاكاة مونت كارلو. تم استخدام برمجية R لأغراض الحسابات الرياضية والاحصائية.

الكلمات الافتتاحية: توزيع بير النوع الثاني عشر ؛ المراقبة المشتركة المعجلة من النوع الثاني ؛ تقدير الإمكان الأعظم؛ فترات الثقة؛ تقدير بيز ؛ دالة الخسارة ؛ سلسلة ماركوف مونتي كارلو.